Q1

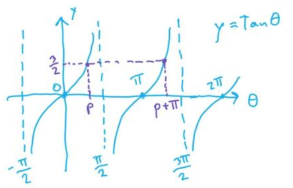
1

Solve the equation $2\sin\theta = 3\cos\theta$ for $0 \le \theta \le 2\pi$, giving your answers to 3 significant figures.

[3]

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$

Essential trigonometric identity!



tan repeats every π radians

$2\sin\theta = 3\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2}$

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 0.982793\ldots = p$ } principal solution

or $\theta = p + \pi = 4.124386\ldots$ } find other solutions in the interval

The solutions are

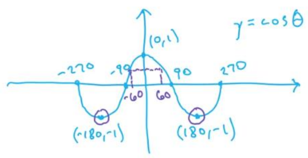
$\boxed{\theta = 0.983 \text{ or } 4.12 \text{ (3 s.f.)}}$

Q2

2

Solve the equation $2\sin^2\theta = \cos\theta + 1$ for $-180° \le \theta \le 180°$.

[5]



$\sin^2\theta + \cos^2\theta \equiv 1 \Rightarrow \sin^2\theta \equiv 1 - \cos^2\theta$

Essential trigonometric identity!

$2\sin^2\theta = \cos\theta + 1$

$2(1 - \cos^2\theta) = \cos\theta + 1$

$2 - 2\cos^2\theta = \cos\theta + 1$

$2\cos^2\theta + \cos\theta - 1 = 0$ "hidden quadratic"

Let $y = \cos\theta$

$2y^2 + y - 1 = (2y-1)(y+1) = 0$

$y = \cos\theta = \frac{1}{2}$ or -1

If $\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}(\frac{1}{2}) = 60°$ principal solution

or $\theta = -60°$ use symmetry to find other solutions in the interval

If $\cos\theta = -1$

$\theta = \cos^{-1}(-1) = 180°$ principal solution

or $\theta = -180°$ use symmetry to find other solutions in the interval

The solutions are

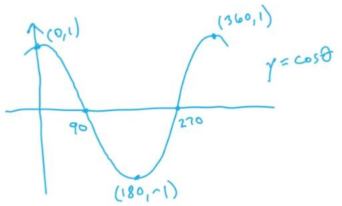
$\boxed{\theta = -180°, -60°, 60°, 180°}$

Q3

3

Given that the angle θ is obtuse and that $\sin\theta = \frac{3}{4}$, find the exact value of $\cos\theta$.

[3]



$\sin^2\theta + \cos^2\theta \equiv 1$ Essential trigonometric identity!

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{4}\right)^2 + \cos^2\theta = 1$

$\frac{9}{16} + \cos^2\theta = 1$

$\cos^2\theta = \frac{7}{16}$

$\cos\theta = \pm\frac{\sqrt{7}}{4}$ Two possible answers!

But for θ between $90°$ and $180°$ (obtuse), cosine is negative

$$\boxed{\cos\theta = -\frac{\sqrt{7}}{4}}$$

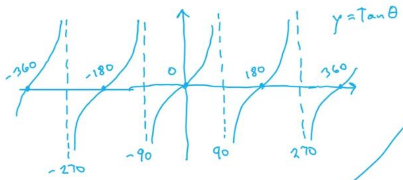
keep $\sqrt{7}$ as exact value!

Q4

4

Solve the equation $\tan 2x = \frac{3}{\tan 2x}$ for $-180° \le x \le 180°$.

[5]



tan repeats every 180°

$-180° \le x \le 180° \implies -360° \le 2x \le 360°$ convert the solution interval

Solve in terms of $2x$, in the converted interval:

$\tan 2x = \frac{3}{\tan 2x}$

$\tan^2 2x = 3 \implies \tan 2x = \pm\sqrt{3}$

If $\tan 2x = \sqrt{3}$

$2x = \tan^{-1}(\sqrt{3}) = 60°$ principal value

$60+180=240$ $60-180=-120$ $-120-180=-300$

So $2x = -300, -120, 240$ are also solutions   find other solutions in interval

If $\tan 2x = -\sqrt{3}$

$2x = \tan^{-1}(-\sqrt{3}) = -60°$ principal value

$-60-180=-240$ $-60+180=120$ $120+180=300$

So $2x = -240, 120, 300$ are also solutions   find other solutions in interval

Dividing those solutions by 2 gives   Only convert to x at the VERY end!

$$\boxed{x = -150°, -120°, -60°, -30°, 30°, 60°, 120°, 150°}$$

Q5

5

Solve the equation $2\tan x - \sin x = 0$ for $-\pi \le x \le \pi$.

[5]

$\tan x \equiv \dfrac{\sin x}{\cos x}$

Essential trigonometric identity!

$2\tan x - \sin x = 0$

$\dfrac{2\sin x}{\cos x} - \sin x = 0$

$\sin x\left(\dfrac{2}{\cos x} - 1\right) = 0$ } factorise

So $\sin x = 0$

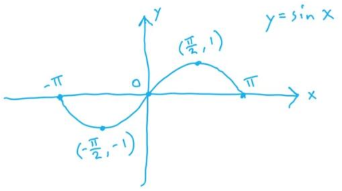
or $\dfrac{2}{\cos x} - 1 = 0 \Rightarrow \dfrac{2}{\cos x} = 1 \Rightarrow \cos x = 2$ No solutions

cos x can never be greater than one!

$\underline{\sin x = 0}$

$x = \sin^{-1}(0) = 0$ } principal value

or $x = -\pi$ or π } find other solutions in interval



The answers are

$\boxed{x = -\pi,\ 0,\ \pi}$

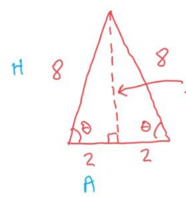
Q6

6

An isosceles triangle has sides 8 cm, 8 cm and 4 cm and equal base angles θ.

Find exact values for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

[6]



$\sqrt{8^2 - 2^2} = \sqrt{60} = 2\sqrt{15}$

Pythagoras' Theorem

$\sin\theta = \dfrac{2\sqrt{15}}{8} = \dfrac{\sqrt{15}}{4}$ SOH

$\cos\theta = \dfrac{2}{8} = \dfrac{1}{4}$ CAH

$\tan\theta = \dfrac{2\sqrt{15}}{2} = \sqrt{15}$ TOA

Q7a

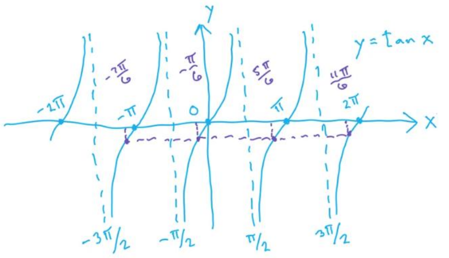
7a

(a) Find all the solutions to the equation $\sqrt{3}\tan 2\theta = -1$ in the interval $-\pi \le \theta \le \pi$, giving your answers in radians as multiples of π.

[4]

(b) Find all the solutions to the equation $6\sin^2 x + 7\sin x - 3 = 0$ in the interval $0 \le x \le 2\pi$, giving your answers in radians to three significant figures.

[5]



Note : repeats every π radians (180°)

a) Start by transforming solution interval:

$-\pi \le \theta \le \pi \Rightarrow -2\pi \le 2\theta \le 2\pi$

Then find all solutions in terms of 2θ:

$\sqrt{3}\tan 2\theta = -1 \Longrightarrow \tan 2\theta = -\frac{1}{\sqrt{3}}$

$2\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$ } principal value

or $2\theta = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$

or $2\theta = \frac{5\pi}{6} + \pi = \frac{11\pi}{6}$

or $2\theta = -\frac{\pi}{6} - \pi = -\frac{7\pi}{6}$

} use symmetry of tangent graph to find other solutions in the transformed interval

At the VERY end, convert into solutions in terms of θ:

Dividing those solutions by 2 gives

$$\boxed{\theta = -\frac{7\pi}{12},\ -\frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{11\pi}{12}}$$

Q7b

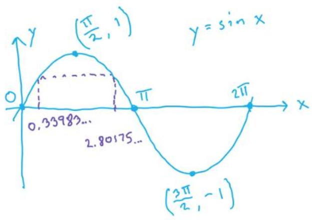
7b

(a) Find all the solutions to the equation $\sqrt{3}\tan 2\theta = -1$ in the interval $-\pi \le \theta \le \pi$, giving your answers in radians as multiples of π.

[4]

(b) Find all the solutions to the equation $6\sin^2 x + 7\sin x - 3 = 0$ in the interval $0 \le x \le 2\pi$, giving your answers in radians to three significant figures.

[5]



b) $6\sin^2 x + 7\sin x - 3 = 0$ "hidden quadratic"

$(2\sin x + 3)(3\sin x - 1) = 0$

$\underbrace{\sin x = -\frac{3}{2}}_{\text{no solutions}}$ or $\sin x = \frac{1}{3}$

$-1 \le \sin x \le 1$ for all values of x

If $\sin x = \frac{1}{3}$, then

$x = \sin^{-1}\left(\frac{1}{3}\right) = 0.33983...$ } principal value

use symmetry of sine function to find other solutions in interval:

or $x = \pi - 0.33983... = 2.80175...$

The solutions are (to 3 s.f.)

$$\boxed{x = 0.340 \text{ or } x = 2.80}$$

Q8a

8a

(a) Show that $x = \frac{1}{2}$ satisfies the equation $8x^3 - 4x^2 - 6x + 3 = 0$. [1]

(b) Hence solve the equation $8\cos^3 x - 4\cos^2 x - 6\cos x + 3 = 0$ for $0° \le x \le 360°$. [6]

a)

$$\boxed{\begin{aligned} &8\left(\tfrac{1}{2}\right)^3 - 4\left(\tfrac{1}{2}\right)^2 - 6\left(\tfrac{1}{2}\right) + 3 \\ &= 8\left(\tfrac{1}{8}\right) - 4\left(\tfrac{1}{4}\right) - 6\left(\tfrac{1}{2}\right) + 3 \\ &= 1 - 1 - 3 + 3 \\ &= 0 \end{aligned}}$$

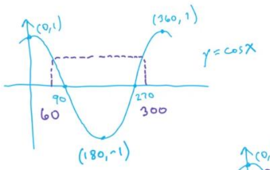
Q8b

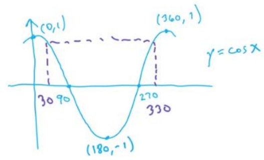
8b

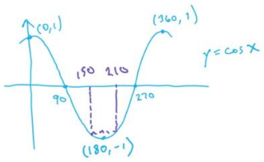
(a) Show that $x = \frac{1}{2}$ satisfies the equation $8x^3 - 4x^2 - 6x + 3 = 0$.

↳ $(2x-1)$ is a factor (Factor Theorem) [1]

(b) Hence solve the equation $8\cos^3 x - 4\cos^2 x - 6\cos x + 3 = 0$ for $0° \le x \le 360°$. [6]







b) $8\cos^3 x - 4\cos^2 x - 6\cos x + 3 = 0$ } a cubic equation in cos x

Let $y = \cos x$

$8y^3 - 4y^2 - 6y + 3 = (2y-1)(4y^2-3) = 0$

$(2y-1)(2y-\sqrt{3})(2y+\sqrt{3}) = 0$

$y = \cos x = 1/2,\ \sqrt{3}/2,\ -\sqrt{3}/2$ solve for cos x

If $\cos x = 1/2$

$x = \cos^{-1}(1/2) = 60°$ principal solution

or $x = 360 - 60 = 300°$ use symmetry to find other solutions in the interval

If $\cos x = \sqrt{3}/2$

$x = \cos^{-1}(\sqrt{3}/2) = 30°$ principal solution

or $x = 360 - 30 = 330°$ use symmetry to find other solutions in the interval

If $\cos x = -\sqrt{3}/2$

$x = \cos^{-1}(-\sqrt{3}/2) = 150°$ principal solution

or $x = 360 - 150 = 210°$ use symmetry to find other solutions in the interval

The solutions are

$x = 30°, 60°, 150°, 210°, 300°, 330°$

Q9a

9a

(a) A seagull sits on the surface of the sea and moves up and down as waves pass.

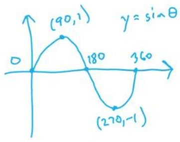
Its height, h metres, above its position in calm water is modelled by the function $h = \frac{2}{5}\sin(180t)°$ where t is the time in seconds after timing commenced.

Find the first time the seagull is 0.3 metres above its calm water position.
Give your answer to 2 decimal places.

[4]

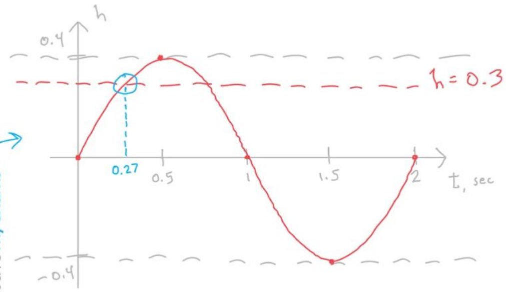
(b) How many times in the first minute after timing commences is the seagull 0.3 metres above its calm water position?

[2]



Note: Sketching the graph is not essential here. It is helpful, however, and will help with part (b) as well.

a) Period $= \frac{360}{180} = 2$ seconds



$0.4\sin(180t) = 0.3$

$\sin(180t) = 0.75$

$180t = \sin^{-1}(0.75) = 48.590377...$

$\boxed{t = 0.27 \text{ sec } (2 \text{ d.p.})}$

Q9b

9b

(a) A seagull sits on the surface of the sea and moves up and down as waves pass.

Its height, h metres, above its position in calm water is modelled by the function $h = \frac{2}{5}\sin(180t)°$ where t is the time in seconds after timing commenced.

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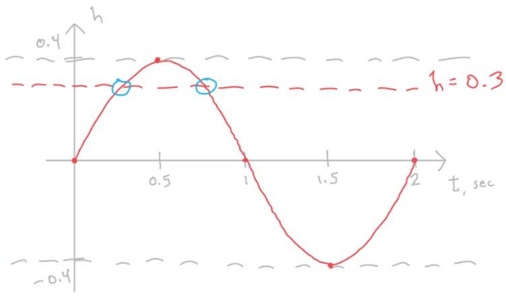

b) In 2 seconds the seagull is at $h = 0.3$ twice.

What happens in the first 2 seconds will repeat 30 times in the first minute (=60 seconds)

$2 \times 30 = 60$

In the first minute, the seagull will be at $h = 0.3$ 60 times.